

# Updated $S_3$ Model of Quarks

Ernest Ma<sup>1</sup> and Blaženka Melić<sup>1,2</sup>

<sup>1</sup> *Department of Physics and Astronomy, University of California,  
Riverside, California 92521, USA*

<sup>2</sup> *Theoretical Physics Division, Rudjer Bošković Institute,  
10002 Zagreb, Croatia*

## Abstract

A model proposed in 2004 using the non-Abelian discrete symmetry  $S_3$  for understanding the flavor structure of quarks and leptons is updated, with special focus on the quark and scalar sectors. We show how the approximate residual symmetries of this model explain both the pattern of the quark mixing matrix and why the recently observed particle of 126 GeV at the Large Hadron Collider is so much like the one Higgs boson of the Standard Model. We identify the strongest phenomenological bounds on the scalar masses of this model, and predict a possibly observable decay  $b \rightarrow s\tau^-\mu^+$ , but not  $b \rightarrow s\tau^+\mu^-$ .

# 1 Introduction

With the discovery [1, 2] of a particle of 126 GeV at the Large Hadron Collider (LHC) and no evidence for any others in a wide range of masses, extensions of the standard model are now severely constrained. In particular, if we want to understand the pattern of quark and lepton masses and their mixing in terms of a flavor symmetry, we are faced with a new theoretical challenge. In order to carry the flavor symmetry in the context of a renormalizable theory, new scalar multiplets are required. We now must have a good reason within the flavor model as to why the one observed light Higgs boson is so much like that of the Standard Model (SM). Of course we need also to understand within the same context of why the quark mixing matrix is nearly diagonal, whereas the neutrino (lepton) mixing matrix is close to tribimaximal.

In 2004, the family structure of quarks and leptons was explained in a model [3] using the non-Abelian discrete symmetry  $S_3$ . It has a symmetry breaking pattern designed to allow for small 2 – 3 mixing in the quark sector and near-maximal 2 – 3 mixing in the neutrino sector. Whereas all the basic details were described for both quarks and leptons, that paper dealt mainly with neutrino mixing, with the specific assumption of negligible  $e - \mu$  mixing in the charged-lepton mass matrix, although this 1 – 2 mixing is generally allowed by the  $S_3$  symmetry and is unavoidable also in the quark sector. As a result of that arbitrary assumption, the neutrino mixing angle  $\theta_{13}$  was predicted to be very small:  $0.02 \pm 0.01$ . Given that it has now been measured [4, 5] at about 0.16, this prediction based on that arbitrary assumption is certainly ruled out, and the observed value of  $\theta_{13}$  should be attributed to  $e - \mu$  mixing within the context of this model. The Higgs sector of this leptonic model has also been studied [6, 7] for its collider signatures.

In this paper we study the quark sector itself in detail and show how the Higgs sector

is constrained by present data. In particular, we identify two approximate residual discrete  $Z_2$  symmetries, which allow the lightest scalar particle to be the observed 126 GeV particle, with the property that it is naturally very close to that of the SM.

In Sec. 2 we present the  $S_3$  model of Ref. [3], writing down specifically all the quark and Higgs representations. In Sec. 3 we discuss the  $c - t$  and  $s - b$  quark sectors and show how they align in a symmetry limit, and the resulting phenomenological constraint from these two sectors. In Sec. 4 we add the  $u$  and  $d$  quarks and discuss the full  $3 \times 3$  quark mixing matrix. In Sec. 5 we consider the full scalar sector consisting of three Higgs doublets, and show how the one light neutral scalar of this sector resembles that of the standard model to a very good approximation, not being a result of fine tuning but based on symmetry. In Sec. 6 we derive the phenomenological constraint on the third Higgs doublet which is responsible for mixing the first family with the other two. In Sec. 7 we make a specific verifiable prediction of this model, i.e.  $b \rightarrow s\tau^-\mu^+$  could have a branching fraction as large as  $10^{-7}$ , whereas  $b \rightarrow s\tau^+\mu^-$  would be suppressed by a relative factor of  $m_\mu^2/m_\tau^2$ . In Sec. 8 we conclude.

## 2 The $S_3$ Model

The smallest non-Abelian discrete symmetry is the group  $S_3$  of the permutation of three objects. It has six elements, and is isomorphic to the symmetry group of the equilateral triangle (identity, rotations by  $\pm 2\pi/3$ , and three reflections). It has three irreducible representations  $\underline{1}, \underline{1}', \underline{2}$ , with the multiplication rules:

$$\underline{1} \times \underline{1}' = \underline{1}', \quad \underline{1}' \times \underline{1}' = \underline{1}, \quad \underline{2} \times \underline{1} = \underline{2}, \quad \underline{2} \times \underline{1}' = \underline{2}, \quad (1)$$

$$\underline{2} \times \underline{2} = \underline{1} + \underline{1}' + \underline{2}. \quad (2)$$

The specific choice of  $2 \times 2$  matrices for the  $\underline{2}$  representation is only unique up to a unitary transformation. A particular practical and elegant one appeared in Ref. [8] which was

followed in Ref. [3]. For a review, see Ref. [9]. In this representation, if

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \sim \underline{2}, \quad (3)$$

under  $S_3$ , then

$$\begin{pmatrix} a_2^\dagger \\ a_1^\dagger \end{pmatrix}, \begin{pmatrix} b_2^\dagger \\ b_1^\dagger \end{pmatrix} \sim \underline{2}; \quad (4)$$

so that

$$a_1 b_2 + a_2 b_1 \sim \underline{1}, \quad a_1 b_2 - a_2 b_1 \sim \underline{1}', \quad \begin{pmatrix} a_2 b_2 \\ a_1 b_1 \end{pmatrix} \sim \underline{2}, \quad (5)$$

and

$$a_1^\dagger b_1 + a_2^\dagger b_2 \sim \underline{1}, \quad a_1^\dagger b_1 - a_2^\dagger b_2 \sim \underline{1}', \quad \begin{pmatrix} a_1^\dagger b_2 \\ a_2^\dagger b_1 \end{pmatrix} \sim \underline{2}. \quad (6)$$

The consequence of this representation is the elegant result that the trilinear combination  $a_1 b_1 c_1 + a_2 b_2 c_2$  is a singlet.

Consider all quarks as left-handed fields, so that the usual right-handed ones are represented by charge conjugates. Let  $Q_i = (u_i, d_i)$ , then we assign as in Ref. [3]

$$Q_1, u^c, d^c, c^c, s^c \sim \underline{1}, \quad t^c, b^c \sim \underline{1}', \quad \begin{pmatrix} Q_2 \\ Q_3 \end{pmatrix} \sim \underline{2}. \quad (7)$$

In analogy to the three quark families, there are also three Higgs doublets  $\Phi_i = (\phi_i^0, \phi_i^-)$  with assignments:

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \sim \underline{2}, \quad \Phi_3 \sim \underline{1}. \quad (8)$$

### 3 The $c - t$ and $s - b$ Quark Sectors

In this sector, only  $\Phi_{1,2}$  are involved in the Yukawa interactions, i.e.

$$\begin{aligned} -\mathcal{L}_Y &= g_1^d [(\phi_1^0 b + \phi_2^0 s) - (\phi_1^- t + \phi_2^- c)] s^c + g_2^d [(\phi_1^0 b - \phi_2^0 s) - (\phi_1^- t - \phi_2^- c)] b^c \\ &+ g_1^u [(\bar{\phi}_2^0 t + \bar{\phi}_1^0 c) + (\phi_2^+ b + \phi_1^+ s)] c^c + g_2^u [(\bar{\phi}_2^0 t - \bar{\phi}_1^0 c) + (\phi_2^+ b - \phi_1^+ s)] t^c. \end{aligned} \quad (9)$$

Consider now the Higgs potential of  $\Phi_{1,2}$ . In addition to the  $S_3$  symmetrical term, we add a soft term which breaks  $S_3$ , but preserves the discrete  $Z_2$  symmetry  $\Phi_1 \leftrightarrow \Phi_2$ . Hence,

$$\begin{aligned} V_{12} = & \mu_1^2(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2) - \mu_2^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) \\ & + \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2)^2 + \frac{1}{2}\lambda_2(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1). \end{aligned} \quad (10)$$

This has a minimum with  $v_1 = v_2 = v = 123$  GeV, where

$$\mu_1^2 - \mu_2^2 + (2\lambda_1 + \lambda_3)v^2 = 0. \quad (11)$$

As a result, the normalized physical scalar bosons and their masses are given by

$$h^0 = \text{Re}(\phi_1 + \phi_2), \quad m^2(h^0) = 2(2\lambda_1 + \lambda_3)v^2, \quad (12)$$

$$H^0 = \text{Re}(\phi_1 - \phi_2), \quad m^2(H^0) = 2\mu_2^2 + 2(2\lambda_2 - \lambda_3)v^2, \quad (13)$$

$$A = \text{Im}(\phi_1 - \phi_2), \quad m^2(A) = 2\mu_2^2, \quad (14)$$

$$H^\pm = \frac{1}{\sqrt{2}}(\phi_1^\pm - \phi_2^\pm), \quad m^2(H^\pm) = 2\mu_2^2 - 2\lambda_3v^2. \quad (15)$$

Note that without the  $\mu_2^2$  term which breaks  $S_3$ ,  $A$  would be massless.

Consider now the generic structure of the  $c - t$  and  $s - b$  mass matrices. They are of the form given by Eq. (12) in Ref. [3], i.e.

$$\mathcal{M} = \begin{pmatrix} g_1v & -g_2v \\ g_1v & g_2v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} g_1\sqrt{2}v & 0 \\ 0 & g_2\sqrt{2}v \end{pmatrix}. \quad (16)$$

Since they are both diagonalized by the same  $2 \times 2$  unitary matrix, there is no mismatch, and the quark mixing matrix is diagonal, explaining to a good first approximation what is observed. Note that this result is based on the symmetry breaking pattern  $S_3 \rightarrow Z_2$ . Now  $h^0$  may be identified with the corresponding SM Higgs boson which couples to  $(m_s/2v)\bar{s}s$  and  $(m_b/2v)\bar{b}b$ , etc.

As for  $H^\pm, H^0, A$ , their Yukawa couplings are given by

$$\mathcal{L}_Y = \frac{m_s}{\sqrt{2}v} \left[ H^+ \bar{t}_L + \left( \frac{H^0 + iA}{\sqrt{2}} \right) \bar{b}_L \right] s_R$$

$$+ \frac{m_b}{\sqrt{2}v} \left[ H^+ \bar{c}_L + \left( \frac{H^0 + iA}{\sqrt{2}} \right) \bar{s}_L \right] b_R + h.c., \quad \text{etc.} \quad (17)$$

Hence  $H^0$  and  $A$  will contribute significantly to  $B_s - \bar{B}_s$  mixing, with resulting bounds on their masses. The tree-level effective four-fermion interaction is given by

$$\frac{m_b^2}{4v^2} \left( \frac{1}{m_H^2} - \frac{1}{m_A^2} \right) (\bar{s}_L b_R)^2 + \frac{m_s m_b}{4v^2} \left( \frac{1}{m_H^2} + \frac{1}{m_A^2} \right) (\bar{s}_L b_R)(\bar{s}_R b_L). \quad (18)$$

It adds to the usual SM box-diagram contribution for  $B_s - \bar{B}_s$  mixing. The two new effective four-quark operators are usually denoted by  $\mathcal{O}_2$  and  $\mathcal{O}_4$  [10], i.e.

$$\mathcal{O}_2 = (\bar{s}_L b_R)^2, \quad (19)$$

$$\mathcal{O}_4 = (\bar{s}_L b_R)(\bar{s}_R b_L). \quad (20)$$

The matrix elements of the operators are calculated at the  $m_{H,A}$  mass scale and evolved to the hadronic scale by using the anomalous dimension matrices given in Ref. [11, 12, 13]. The mass difference in the  $B_s - \bar{B}_s$  system is then given by

$$\Delta M_s = (\Delta M_s)_{\text{SM}} + (\Delta M_s)_{\mathcal{O}_2} + (\Delta M_s)_{\mathcal{O}_4}, \quad (21)$$

where

$$(\Delta M_{B_s})_{\mathcal{O}_2} = 2 \frac{1}{2m_{B_s}} \frac{m_b^2}{4v^2} \left( \frac{1}{m_H^2} - \frac{1}{m_A^2} \right) \eta_2(\mu_b) \left( -\frac{5}{12} \left( \frac{m_{B_s}}{m_b(\mu_b) + m_s(\mu_b)} \right)^2 m_{B_s}^2 f_{B_s}^2 \right) B_2(\mu_b), \quad (22)$$

$$(\Delta M_{B_s})_{\mathcal{O}_4} = 2 \frac{1}{2m_{B_s}} \frac{m_b m_s}{4v^2} \left( \frac{1}{m_H^2} + \frac{1}{m_A^2} \right) \eta_4(\mu_b) \left( \frac{1}{2} \left( \frac{m_{B_s}}{m_b(\mu_b) + m_s(\mu_b)} \right)^2 m_{B_s}^2 f_{B_s}^2 \right) B_4(\mu_b). \quad (23)$$

For the bag model parameters we use the results from Ref. [14] estimated in the quenched approximation on the lattice ( $\mu_b = m_b^{\text{RI-MOM}} = 4.6 \text{ GeV}$ ):

$$B_2(\mu_b) = 0.82, \quad B_4(\mu_b) = 1.16, \quad (24)$$

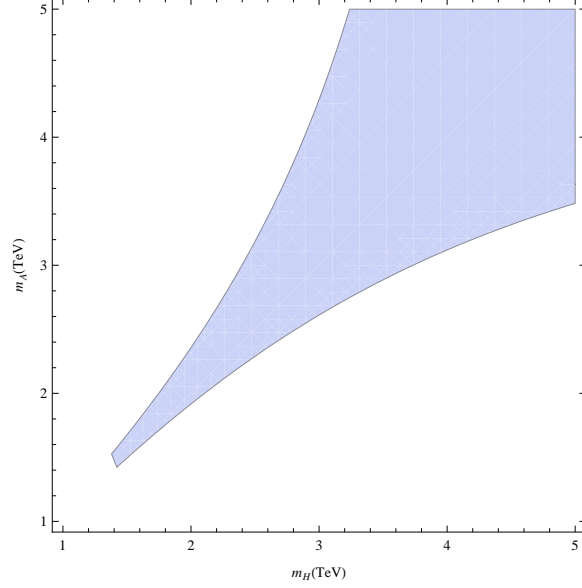


Figure 1:  $B_s - \bar{B}_s$  mixing constraint for the masses  $m_H$  and  $m_A > 1$  TeV. Masses less than 1 TeV require fine tuning to satisfy the constraint.

and the running of Wilson coefficients given at the same scale [14]:  $\eta_2(\mu_b) \simeq 2.03$ ,  $\eta_4(\mu_b) \simeq 3.23$ . The masses of the  $b$  quark and the  $s$  quark at the TeV scale are  $m_b = 2.4$  GeV and  $m_s = 45$  MeV, respectively.

The current experimental value of  $\Delta m_{B_s}$  is  $116.4 \pm 0.5 \times 10^{-10}$  MeV [15], which agrees with the SM prediction to about 10%. Hence we limit our new physics contribution to  $11.6 \times 10^{-10}$  MeV. The allowed range for the masses of  $H$  and  $A$  Higgs bosons in the region greater than 1 TeV is shown in Fig. 1. Smaller masses, although possible, require fine tuning to satisfy the experimental  $B_s - \bar{B}_s$  bound.

Note that in this approximation, there is no contribution to  $b \rightarrow s\gamma$  from the new scalars, because of a residual  $Z_2$  discrete symmetry under which  $(c, s)$  are even, whereas  $(H^\pm, H^0, A)$  and  $(t, b)$  are odd.

## 4 The $3 \times 3$ Quark Mixing Matrix

The introduction of the third Higgs doublet  $\Phi_3$  will allow the  $u$  and  $d$  quarks to obtain mass, as well as mixing with the other quarks. Since these are all small, it is natural to assume that  $v_3$  is small. (This also means that the  $\Phi_3$  mass may be naturally large, as shown later.) In that case, the  $h^0$  of Eq. (12) is still a very good approximation to the one physical Higgs boson  $h_{SM}^0$  of the SM, i.e. we have an explanation of why our specific three-Higgs doublet model has a mass eigenstate  $h^0$  which is very close to  $h_{SM}^0$ .

The  $3 \times 3$  quark mass matrices are given by [3]

$$\mathcal{M}_d = \begin{pmatrix} g_3^d v_3 & g_4^d v_3 & 0 \\ 0 & g_1^d v_2 & -g_2^d v_2 \\ 0 & g_1^d v_1 & g_2^d v_1 \end{pmatrix}, \quad (25)$$

and

$$\mathcal{M}_u = \begin{pmatrix} g_3^u v_3^* & g_4^u v_3^* & 0 \\ 0 & g_1^u v_1^* & -g_2^u v_1^* \\ 0 & g_1^u v_2^* & g_2^u v_2^* \end{pmatrix}. \quad (26)$$

Note that  $v_1, v_2$  in  $\mathcal{M}_d$  are replaced by  $v_2^*, v_1^*$  in  $\mathcal{M}_u$ . This means that for  $v_1 \neq v_2$ , there will be a mismatch in the  $s - b$  and  $c - t$  sectors. Since  $m_s \ll m_b$  and  $m_c \ll m_t$ , these mass matrices are simply diagonalized on the left:  $\mathcal{M}_d$  by

$$V_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c' & -s' \\ 0 & s' & c' \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (27)$$

where  $s'/c' = v_2/v_1$ , and  $\mathcal{M}_u$  by

$$V_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & s' & -c' \\ 0 & c' & s' \end{pmatrix} \begin{pmatrix} c_u & -s_u e^{i\delta} & 0 \\ s_u e^{-i\delta} & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (28)$$

We have rephased  $d_R, s_R, b_R, u_R, c_R, t_R$  as well as  $(u, d)_L, (c, s)_L$  so that only one complex phase  $\delta$  remains. Hence

$$V_{CKM} = V_u^\dagger V_d = \begin{pmatrix} c_u & s_u e^{i\delta} & 0 \\ -s_u e^{-i\delta} & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c'' & s'' \\ 0 & -s'' & c'' \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$$= \begin{pmatrix} c_u c_d + c'' s_u s_d e^{i\delta} & -c_u s_d + c'' s_u c_d e^{i\delta} & s'' s_u e^{i\delta} \\ -s_u c_d e^{-i\delta} + c'' c_u s_d & s_u s_d e^{-i\delta} + c'' c_u c_d & s'' c_u \\ -s'' s_d & -s'' c_d & c'' \end{pmatrix}, \quad (29)$$

where  $s''/c'' = (c'^2 - s'^2)/2s'c'$ . Using the 2012 Particle Data Group values [15], we obtain

$$s'' = 0.04135, \quad s_u = 0.08489, \quad s_d = 0.20983, \quad \cos \delta = -5.47 \times 10^{-3}, \quad (30)$$

with the CP violating parameter

$$J = s_u c_u s_d c_d (s'')^2 c'' \sin \delta = 2.96 \times 10^{-5}. \quad (31)$$

## 5 The Complete Higgs Sector

To allow for  $v_1 \neq v_2$ , the symmetry  $\Phi_1 \leftrightarrow \Phi_2$  must be broken. This may be accomplished by adding to  $V_{12}$  of Eq. (10) the term  $\mu_3^2(\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2)$ . The coefficient  $\mu_3^2$  may be chosen naturally small, because  $\mu_3^2 = 0$  results in the extra  $Z_2$  symmetry already discussed. In addition, we impose a new  $Z_2$  symmetry so that  $\Phi_3$  and  $(u, d)_L$  are odd and all other fields are even on  $Z_2$ . The purpose of this symmetry is to forbid the quartic term  $\Phi_3^\dagger(\Phi_1 \Phi_2^\dagger \Phi_1 + \Phi_2 \Phi_1^\dagger \Phi_2) + h.c.$  which is allowed by  $S_3$  (the reason for this will become clear later). However, we also allow this new  $Z_2$  symmetry to be broken softly by the bilinear term  $\mu_4^2 \Phi_3^\dagger(\Phi_1 + \Phi_2) + h.c.$ , which preserves the  $\Phi_1 \leftrightarrow \Phi_2$  interchange symmetry of  $V_{12}$ . The complete scalar potential of this model is then

$$\begin{aligned} V_{123} &= V_{12} + \mu_3^2(\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2) + m_3^2 \Phi_3^\dagger \Phi_3 + [\mu_4^2 \Phi_3^\dagger(\Phi_1 + \Phi_2) + h.c.] \\ &+ \frac{1}{2} \lambda_4 (\Phi_3^\dagger \Phi_3)^2 + \lambda_5 (\Phi_3^\dagger \Phi_3)(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2) + \lambda_6 \Phi_3^\dagger(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2) \Phi_3 \\ &+ [\lambda_7 (\Phi_3^\dagger \Phi_1)(\Phi_3^\dagger \Phi_2) + h.c.]. \end{aligned} \quad (32)$$

Consider first  $v_1 \neq v_2$ , this results in a deviation of  $h^0$  from  $h_{SM}^0$  given by

$$h^0 - h_{SM}^0 \simeq \frac{(\lambda_1 - \lambda_2 + \lambda_3)(v_1^2 - v_2^2)}{2\mu_2^2} H^0, \quad (33)$$

where  $(v_1^2 - v_2^2)/4v^2 = 0.0207$  from  $s'' = 0.04135$  of Eq. (23).

Consider now  $v_3 \neq 0$ . Since  $m_3^2 > 0$  is assumed large,

$$v_3 \simeq \frac{-\mu_4^2(v_1 + v_2)}{m_3^2 + (\lambda_5 + \lambda_6)(v_1^2 + v_2^2) + 2\lambda_7 v_1 v_2} \simeq \frac{-2v\mu_4^2}{m_3^2}, \quad (34)$$

and the mixing of  $\phi_{3R}$  with  $\phi_{1R} + \phi_{2R}$  is  $v_3/(2v)$ . This results in

$$h^0 - h_{SM}^0 \simeq \frac{v_3 m_h^2}{2v m_3^2} \text{Re}(\phi_3^0). \quad (35)$$

The  $h^0$  of our model is thus naturally equal to  $h_{SM}^0$  in the symmetry limit  $v_1 = v_2$  and  $v_3 = 0$ , and the deviation is naturally suppressed for realistic values of  $v_3^2 < v_2^2 \simeq v_1^2$ . If the quartic scalar term forbidden by the new  $Z_2$  symmetry exists, then  $h^0 - h_{SM}^0 \simeq (2v_3/v)\text{Re}(\phi_3^0)$  would replace Eq. (30). This means that  $h^0$  exchange itself will contribute too much to  $K^0 - \bar{K}^0$  mixing. With the relation in Eq. (30), this contribution is negligible.

## 6 Constraint on $\Phi_3$

The exchange of  $\phi_3^0$  directly contributes to  $\Delta M_K$ . The relevant effective interaction is given by

$$\frac{s_d^2 c_d^2 m_d m_s}{v_3^2 m_3^2} (\bar{d}_L s_R)(\bar{d}_R s_L). \quad (36)$$

This operator is again  $\mathcal{O}_4$  from Eq. (20), with the appropriate exchange of the quarks and the contribution to  $\Delta m_K$  is analogous to Eq. (23), i.e.

$$(\Delta M_K)_{\mathcal{O}_4} = 2 \frac{1}{2m_K} s_d^2 c_d^2 \frac{m_s m_d}{v_3^2} \frac{1}{m_3^2} \eta_{4K}(\mu_K) \left( \frac{1}{2} \left( \frac{m_K}{m_s(\mu_K) + m_d(\mu_K)} \right)^2 m_K^2 f_K^2 \right) B_{4K}(\mu_K), \quad (37)$$

with  $\mu_K = 2$  GeV. The relevant parameters are taken from Ref. [13]:

$$B_{4K}^{\text{eff}}(\mu_K) = \frac{1}{2} \left( \frac{m_K}{m_s(\mu_K) + m_d(\mu_K)} \right)^2 B_{4K}(\mu_K) = 19.31, \quad (38)$$

where  $B_{4K}(\mu_K) = 1.03$  itself, and the running of the Wilson coefficient gives  $\eta_{4K}(\mu_K) = 4.87$  [16].

Assuming that this contribution is no more than 20% of the experimental value  $\Delta M_K = 3.483 \pm 0.006 \times 10^{-12}$  MeV [15], we obtain

$$v_3 m_3 > 6.0 \times 10^4 \text{ GeV}^2. \quad (39)$$

If  $v_3 = 10$  GeV, then  $m_3 > 6$  TeV.

So far, we have been able to show that  $h^0$  is very close to  $h_{SM}^0$ , and that the other physical scalars are of order 1 to 10 TeV, from  $B_s - \bar{B}_s$  and  $K^0 - \bar{K}^0$  mixing respectively. All other effective flavor-changing neutral-current interactions in the quark sector such as  $b \rightarrow s\gamma$  are suppressed.

## 7 Specific Prediction

Since the scalars of this model also have leptonic interactions, there are some lepton flavor violating processes in this model which are negligible in the SM. The corresponding Lagrangian to Eq. (17) for the  $\mu - \tau$  sector is given by

$$\begin{aligned} \mathcal{L}_Y &= \frac{m_\mu}{\sqrt{2}v} \left[ H^+ \bar{\nu}_{\tau L} + \left( \frac{H^0 + iA}{\sqrt{2}} \right) \bar{\tau}_L \right] \mu_R \\ &+ \frac{m_\tau}{\sqrt{2}v} \left[ H^+ \bar{\nu}_{\mu L} + \left( \frac{H^0 + iA}{\sqrt{2}} \right) \bar{\mu}_L \right] \tau_R + h.c. \end{aligned} \quad (40)$$

Hence  $b \rightarrow s\tau^-\mu^+$  proceeds again through the exchange of  $H^0 + iA$  (but  $b \rightarrow s\tau^+\mu^-$  is suppressed by  $m_\mu^2/m_\tau^2$ ).

Experimentally the most interesting decay would be  $B_s \rightarrow \tau^+\mu^-$ . The branching ratio for this decay can be written as

$$BR(B_s \rightarrow \tau^+\mu^-) = \frac{m_{B_s}^5 f_{B_s}^2}{64\pi} \tau_{B_s} m_\tau^2 \left( 1 - \frac{m_\tau}{m_{B_s}} \right)^2 \frac{1}{v^4} \left( \frac{1}{m_H^2} + \frac{1}{m_A^2} \right)^2 \quad (41)$$

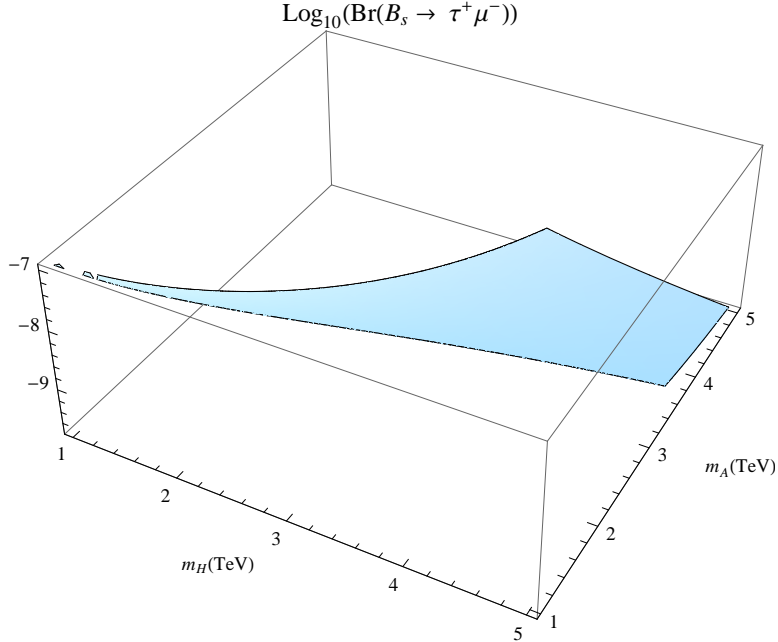


Figure 2: Prediction for  $B_s \rightarrow \tau^+ \mu^-$  lepton flavor violation process in our model, with the masses of  $H$  and  $A$  bosons constrained by the  $B_s - \bar{B}_s$  mixing from Fig.1.

We see in Fig. 2 that the branching ratio for this decay can be as high as  $10^{-7}$  for masses of  $H$  and  $A$  bosons in the TeV range. This is a possible unique signature of this model. For the less realistic masses smaller than 1 TeV, the  $B_s \rightarrow \tau^+ \mu^-$  branching ratio can come up to  $O(10^{-6})$ .

## 8 Conclusion

In the post-Higgs era, any extension of the SM has to face the question of why it contains a light neutral scalar boson  $h^0$  so much like to  $h_{SM}^0$ , in addition to being consistent with a myriad of phenomenological precision measurements. We show how this is possible in a model [3] proposed in 2004 based on the non-Abelian discrete symmetry  $S_3$ . It has three Higgs doublets, and yet one becomes almost exactly that of the SM because of two ap-

proximate residual  $Z_2$  symmetries. It also explains why the quark mixing matrix is nearly diagonal, with just enough parameters to fit the data precisely. The model is constrained principally by  $B_s - \bar{B}_s$  and  $K^0 - \bar{K}^0$  mixing, and has the unique prediction of  $b \rightarrow s\tau^-\mu^+$  with a branching fraction for  $B_s \rightarrow \tau^+\mu^-$  decay as large as  $10^{-7}$ , but a strong suppression by the factor  $m_\mu^2/m_\tau^2$  for  $b \rightarrow s\tau^+\mu^-$  decay.

Acknowledgment : This work is supported in part by the U. S. Department of Energy under Grant No. DE-AC02-06CH11357. BM acknowledges support of the Fulbright foundation.

## References

- [1] ATLAS Collaboration, G. Aad *et al.*, Phys. Lett. **B716**, 1 (2012).
- [2] CMS Collaboration, S. Chatrchyan *et al.*, Phys. Lett. **B716**, 30 (2012).
- [3] S.-L. Chen, M. Frigerio, and E. Ma, Phys. Rev. **D70**, 073008 (2004).
- [4] Daya Bay Collaboration, F. P. An *et al.*, Phys. Rev. Lett. **108**, 171803 (2012).
- [5] RENO Collaboration, J. K. Ahn *et al.*, Phys. Rev. Lett. **108**, 191802 (2012).
- [6] G. Bhattacharyya, P. Leser, and H. Pas, Phys. Rev. **D83**, 011701 (2011).
- [7] G. Bhattacharyya, P. Leser, and H. Pas, Phys. Rev. **D86**, 036009 (2012).
- [8] N. G. Deshpande, M. Gupta, and P. B. Pal, Phys. Rev. **D45**, 953 (1992).
- [9] E. Ma, Mount Fuji Lectures, arXiv:hep-ph/0409075.
- [10] A. J. Buras, J. Girrbach and R. Ziegler, “Particle-Antiparticle Mixing, CP Violation and Rare K and B Decays in a Minimal Theory of Fermion Masses,” arXiv:1301.5498 [hep-ph].

- [11] M. Ciuchini, E. Franco, V. Lubicz, G. Martinelli, I. Scimemi and L. Silvestrini, Nucl. Phys. B **523**, 501 (1998).
- [12] A. J. Buras, M. Misiak and J. Urban, Nucl. Phys. B **586**, 397 (2000).
- [13] A. J. Buras, S. Jager and J. Urban, Nucl. Phys. B **605**, 600 (2001).
- [14] D. Becirevic, M. Ciuchini, E. Franco, V. Gimenez, G. Martinelli, A. Masiero, M. Papinutto and J. Reyes *et al.*, Nucl. Phys. B **634**, 105 (2002).
- [15] Particle Data Group Collaboration, J. Beringer *et al.*, Phys. Rev. D **86** 010001 (2012).
- [16] M. Ciuchini, V. Lubicz, L. Conti, A. Vladikas, A. Donini, E. Franco, G. Martinelli and I. Scimemi *et al.*, JHEP **9810**, 008 (1998).